Transport of magnetoexcitons in single and coupled quantum wells

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Abstract

The transport relaxation time $\tau(P)$ and the mean free path of magnetoexcitons in single and coupled quantum wells are calculated (P is the magnetic momentum of the magnetoexciton). We present the results for magnetoexciton scattering in a random field due to (i) quantum well width fluctuations, (ii) composite fluctuations and (iii) ionized impurities. The time $\tau(P)$ depends nonmonotonously on P in the case (ii) and in the cases (i), (iii) for D/l smaller than some critical value (D is the interwell separation, $l = \sqrt{\hbar c/eH}$ is the magnetic length). For $D/l \gg 1$ the transport relaxation time increases monotonously with P.

The magnetoexciton mean free path $\lambda(P)$ has a maximum at $P \neq 0$ in the cases (i), (iii). It decreases with increasing D/l. The mean free path calculated for the case (ii) may have two maxima. One of them disappears with the variation of the random fields parameters. The maximum of $\lambda(P)$ increases with H for types (i,iii) of scattering processes and decreases in the case (ii).

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I. INTRODUCTION

Electron(e)-hole(h) systems in coupled quantum wells particularly in strong magnetic fields, have attracted considerable interest over the past few years [1-7], long after the prediction of the superfluidity in the system manifesting itself in a sense as a superconductivity in the "electron" and "hole" quantum wells or wires [8] (see also [9,10] and references therein; the magnetic field effects were analyzed in [11-15]). An interesting drag effects in this system was also studied [16,17]. The application of strong transverse magnetic fields increases the effective exciton mass and exciton binding energy [11,12] and thus effect on the conditions of the superfluid phase formation [13].

Indirect excitons or pairs of spatially separated e and h may also condense in a variety of different phases. Some of them are analogous to the phases in the system of bulk excitons [18,19]. The existence of these phases needs sufficiently long exciton life time in comparison with the thermalization time. Experimental investigations [4,6] of the photoluminescense of spatially-separated electron-hole systems in transverse magnetic fields in coupled quantum wells (CQW) discovered dramatical reductions of the lifetimes of indirect magnetoexcitons with increasing magnetic field strength for $B > 7$T and at low temperatures, $T < 1$K. Indirect excitons are characterized by large radiative lifetimes because the spatial confinement of electrons and holes in different quantum wells (QW) suppresses the e-h annihilation rate due to the small overlap of electron and hole wave functions. One can conclude therefore that the total exciton lifetime is determined by the relaxation time of exciton transport to the recombination centers which strongly depends on the magnetic field. Exciton-exciton collisions and indirect-direct exciton conversion induced by the collisions are not essential in the dilute exciton gas.

Two-dimensional (2D) exciton diffusion and localization in random fields at $B = 0$ were considered in [20-23]. The transport properties of the bulk excitons were studied in [24-26]. Low temperature transport of 2D excitons in *double* QWs in the limiting cases of zero magnetic field and magnetic quantum limit is considered in [27]. In the present paper we study the effects of the interface roughness (IFR), impurities in CQWs and local concentration fluctuations in $A^{III}B^{V}$ solid solution on 2D direct and indirect magnetoexciton transport at low temperatures.

We show that the transport momentum relaxation time τ of indirect magnetoexcitons essentially depends on the magnetoexciton momentum $\mathbf{P}$ and interwell separation D between electron and hole QWs. It occurs that the transport momentum relaxation times of direct and indirect magnetoexciton scattering on IFR are nonmonotoneous functions of $\mathbf{P}$ and have maximum τ_{max} at nonzero magnetic momentum and small interwell separation due to the nonmonotonous dependence of the magnetoexciton group velocity and interaction matrix element on $\mathbf{P}$. The calculated transport relaxation time behavior is in qualitative agreement with the experimental data [4] in the limiting case of high magnetic fields.

The organization of this paper is as follows. The transport momentum relaxation time and mean free path of magnetoexciton scattering on IFR in QW and in CQWs are calculated respectively in Sec. II and Sec. IV. In Sec. III we consider magnetoexciton transport in the random field of the local concentration fluctuations in $A^{III}B^{V}$ solid solution. The transport properties of magnetoexcitons in the presence of randomly distributed ionized impurities are considered in Sec. V. Conclusions are presented in Sec. VI.

II. DIRECT MAGNETOEXCITON IFR SCATTERING

The scattering potential for excitons can arise due to the local fluctuations in the QW width and have the form [23,27]

$$V(\mathbf{r}_e, \mathbf{r}_h) = V_e(\mathbf{r}_e) + V_h(\mathbf{r}_h), \tag{1}$$

$$V_{e,h}(\mathbf{r}) = \alpha_{e,h}(\xi_1(\mathbf{r}) - \xi_2(\mathbf{r})),$$

where $\alpha_{e,h} = \frac{\partial E_{e,h}^{(0)}}{\partial d}$, d is the QW width, $E_{e(h)}^{(0)}$ is the energy of the e(h) lowest level in the conduction (valence) band, $\xi_{1,2}(\mathbf{r})$ are local QW width fluctuations on different interfaces. The fluctuations $\xi_1(\mathbf{r})$ and $\xi_2(\mathbf{r})$ are statistically independent, while the fluctuations on the same interface are described by Gaussian autocorrelation functions, i.e.

$$\ll \xi_i(r_1)\xi_j(r_2) \gg = \delta_{ij}\Delta_i^2 \exp\left(-\frac{(r_1 - r_2)^2}{2\Lambda_i^2}\right), \tag{2}$$

where Δ_i is the amplitude of fluctuations ξ_i, Λ_i is the in-plane correlation length. Further we consider the case of high transverse magnetic fields ($l \ll a_{e,h}$; in this case magnetoexciton states can be classified by Landau level quantum numbers n and m [11,12]), thin QWs ($d^2 \ll a_{e,h}l$; in this case the energy of e and h size quantization is much greater than the exciton energies) and small width fluctuations $\Delta_i \ll d$ ($a_{e,h} = \epsilon\hbar^2/m_{e,h}e^2$ are the effective Bohr radii of electron (e) and hole (h), $\epsilon = (\epsilon_1 + \epsilon_2)/2$, $\epsilon_{1,2}$ are the dielectric susceptibilities of the media surrounding e and h QWs). The value Δ_i can have the order of one monolayer height $\approx 3\mathring{A}$ (see [28-30]). The correlation length Λ_i essentially depends on the QW growth method [29]. One can consider two characteristic exciton transport regimes in dependence on the value Λ_i/r_{exc} (r_{exc} is the exciton size in the QW plane layer which is proportional to l in high magnetic fields [11,12]). For $\Lambda_i \approx r_{exc}$ excitons may be localized in minima of local potential fluctuations with average size Λ_i. Hence localized exciton states are formed [28], analogous to states in quantum dots [31] (excitons in 'natural' quantum dots). In this case the low- temperature exciton transport is apparently thermally-activated between states in different minima of random potential [21]. In sections II and IV of the paper we consider the opposite case $\Lambda_i \gg r_{exc}$ which takes place in structures $AlGaAs - GaAs$ with smooth interfaces. (see [28-30]). More strictly the criterium of the interface smoothness in the framework of the exciton problem is

$$r_{exc}\sqrt{\ll \nabla V^2 \gg} \ll E_I, \tag{3}$$

where E_I is the energy of exciton ionization. For $D \ll l$ we get from inequality (3)

$$\Lambda \gg \frac{a_{e,h}^z l^2 \Delta}{d^3}\sqrt{2\pi^3} \tag{4}$$

and for $D \gg l$

$$\Lambda \gg \frac{a_{e,h}^z l D \Delta}{d^3}\sqrt{2\pi^3} \tag{5}$$

($a_{e,h}^z = \epsilon\hbar^2/m_{e,h}^z e^2$ are the effective Bohr radii of e and h in the z-direction).

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The transport momentum relaxation time of magnetoexcitons is calculated in the Born approximation, which is valid in the case of $\hbar^2/M_{exc}\Lambda^2 \gg \Delta\hbar^2/\pi^2 m_{e,h}^z d^3$, i.e.

$$\pi\sqrt{\frac{d^3}{\Delta}\frac{m_{e,h}^z}{M_{exc}}} \gg \Lambda \qquad (6)$$

With D and H increasing the effective magnetoexciton mass M_{exc} increases [11,12]. Hence the upper limit of the correlation length variation decreases. The magnetoexciton energy E_I decreases (increases) with D (H). The lower limit of Λ variation increases (decreases) with $D(H)$.

The magnetoexciton relaxation time in Born approximation is given by [23,26,27]

$$\tau^{-1}(P) = \frac{2\pi}{\hbar}\sum_{P'} \ll |<P'|V(\mathbf{r}_e,\mathbf{r}_h)|P>|^2 \gg (1-cos(\phi_{PP'}))\delta(\mathcal{E}(P)-\mathcal{E}(P')), \qquad (7)$$

where $\mathcal{E}(P)$ is the dispersion law of magnetoexcitons in QW (direct magnetoexciton) in the lowest Landau level ($n = m = 0$) [11]:

$$\mathcal{E}(P) = \frac{1}{2}\hbar\omega_c - \frac{e^2}{\epsilon l}\sqrt{\frac{\pi}{2}}I_0\left(\frac{P^2 l^2}{4\hbar^2}\right)\exp\left(-\frac{P^2 l^2}{4\hbar^2}\right) \qquad (8)$$

where $\omega_c = eH/\mu c$ is the cyclotron energy, $\mu = m_e m_h/(m_e + m_h)$ is the in-plane reduced mass, $\mathbf{P}$ is the magnetoexciton momentum (which is a conserved quantity for an isolated magnetoexciton in an ideal system), $I_n(x)$ are modified Bessel functions. The scattering on IFR between two magnetoexciton states ($n = m = 0, \mathbf{P}$) and ($n = m = 0, \mathbf{P'}$) is described by the matrix element (1)

$$<\mathbf{P'}\mid V(\mathbf{r}_e,\mathbf{r}_h)\mid\mathbf{P}>= \frac{1}{S}\exp\left(-(\mathbf{P'}-\mathbf{P})^2 l^2/4\hbar^2\right)$$

$$\left(V_e(\mathbf{P'}-\mathbf{P})\exp\left(il^2\mathbf{H}[\mathbf{P},\mathbf{P'}]/2\hbar^2 H\right) + V_h(\mathbf{P'}-\mathbf{P})\exp\left(-il^2\mathbf{H}[\mathbf{P},\mathbf{P'}]/2\hbar^2 H\right)\right), \qquad (9)$$

then

$$\ll|<\mathbf{P'}\mid V(\mathbf{r}_e,\mathbf{r}_h)\mid\mathbf{P}>|^2\gg=$$

$$\frac{\pi}{2S}\left(\alpha_e^2 + \alpha_h^2 + 2\alpha_e\alpha_h\cos\left(l^2[\mathbf{P},\mathbf{P'}]/\hbar^2\right)\right)\sum_{i=1,2}\exp\left(-(\mathbf{P'}-\mathbf{P})^2 l^2\beta_i/2\hbar^2\right)(\Delta_i\Lambda_i)^2, \qquad (10)$$

where $\beta_i = 1 + \Lambda_i^2/l^2$.

Using Eqs. (10) and (7) the following expression for the direct magnetoexciton relaxation time is obtained:

$$\tau^{-1}(P) = \tau_e^{-1}(P) + \tau_h^{-1}(P) + \tau_{eh}^{-1}(P) , \qquad (11)$$

where

$$\tau_{e,h}^{-1}(P) = \frac{\pi}{4\hbar^3}\left|\frac{\partial\mathcal{E}(P)}{\partial P^2}\right|^{-1}\alpha_{e,h}^2\sum_{i=1,2}(\Delta_i\Lambda_i)^2\exp(-P^2 l^2\beta_i/\hbar^2)$$

$$\left(I_0(P^2 l^2\beta_i/\hbar^2) - I_1(P^2 l^2\beta_i/\hbar^2)\right) \qquad (12)$$

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and

$$\tau_{eh}^{-1}(P) = \frac{\pi\alpha_e\alpha_h}{2\hbar^3}\left|\frac{\partial\mathcal{E}(P)}{\partial P^2}\right|^{-1}\sum_{i=1,2}(\Delta_i\Lambda_i)^2\exp(-P^2l^2\beta_i/\hbar^2)$$

$$\left(I_0(P^2l^2\sqrt{\beta_i^2-1}/\hbar^2) - \frac{\beta_i}{\sqrt{\beta_i^2-1}}I_1(P^2l^2\sqrt{\beta_i^2-1}/\hbar^2)\right) \tag{13}$$

For $l \ll a_{e,h}$ and $d^2 \ll a_{e,h}l$ transverse motion of e and h along the magnetic field does not lead to qualitative changes of the exciton spectrum [11,12], so analogously to [23] we assume that $\alpha_{e,h} = -\pi^2\hbar^2/m_{e,h}^z d^3$.

We have calculated the transport momentum relaxation time (Fig. 1) of direct excitons at $H = 20$ Oe in $GaAs/Al_xGa_{1-x}As$ QWs using the following values of parameters: $d = 40\mathring{A}$, $\Lambda_1 = 100\mathring{A}$, $\Lambda_2 = 30\mathring{A}$, $\Delta_1 = \Delta_2 = 3\mathring{A}$, e and h effective masses in H-direction are $m_e^z = 1.1(1.3)$ and $m_h^z = 0.75(0.34)$ in $AlAs(GaAs)$ (see Ref. [32]). The function $\tau(P)$ is *nonmonotonous* and has a maximum at $Pl = 2.6\hbar$. It occurs because the scattering probability ($1/\tau$) (due to the density of states in the final state - see Eq. (11,13)) is inversely proportional to the magnetoexciton group velocity

$$\frac{\partial\mathcal{E}(P)}{\partial P^2} = \frac{V_g(P)}{2P} \tag{14}$$

The function $V_g(P)$ has a *maximum* which *corresponds to the saddle point* of $\mathcal{E}(P)$. With H increasing τ_{max} increases and shifts to larger magnetic momenta. Expression (11) has the following asymptotics:

$$\tau(P) = \frac{\hbar e^2l}{\sqrt{2\pi}\epsilon}\frac{1}{(\alpha_e+\alpha_h)^2}\begin{cases}((\Delta_1\Lambda_1)^2 + (\Delta_2\Lambda_2)^2)^{-1}, & P = 0, \\ 8\left(\frac{(\Delta_1\Lambda_1)^2}{\beta_1^{3/2}} + \frac{(\Delta_2\Lambda_2)^2}{\beta_2^{3/2}}\right)^{-1}, & Pl/\hbar \gg 1,\end{cases} \tag{15}$$

The magnetoexciton mean free path is treated as $\lambda(P) = v(P)\tau(P)$ (Fig. 2), where $v(P) = \frac{\partial\mathcal{E}(P)}{\partial P}$. For $P = 0$ we obtain $\lambda(0) = 0$ ($v(0) = 0$) and for $Pl/\hbar \gg 1$ we get

$$\lambda = \frac{4\sqrt{2}\hbar^2e^4}{\sqrt{\pi}\epsilon^2P^2l}\frac{1}{(\alpha_e+\alpha_h)^2}\left(\frac{(\Delta_1\Lambda_1)^2}{\beta_1^{3/2}} + \frac{(\Delta_2\Lambda_2)^2}{\beta_2^{3/2}}\right)^{-1} \tag{16}$$

For $Pl/\hbar \ll 1$ and $Pl/\hbar \gg 1$ the magnetoexciton mean free path is typically smaller than the correlation length of disorder Λ. This situation means magnetoexciton localization [33]. Magnetic field increase leads to increasing λ_{max}.

III. COMPOSITION FLUCTUATIONS IN QW

Random potential in QW can arise due to local concentration fluctuations of the components of QW structure [20,34]. The interaction of excitons with composition fluctuations has the form [35]

$$V(\mathbf{r}_e, \mathbf{r}_h) = \alpha_e\xi(\mathbf{r}_e) - \alpha_h\xi(\mathbf{r}_h), \tag{17}$$

where $\alpha_{e,h} = \frac{1}{N}\frac{\partial E_{e,h}^{(0)}}{\partial x}$, x is the mean relative concentration of A-atoms, N is the concentration of lattice sites where two types of atoms may arrange, $\xi(\mathbf{r})$ is the surplus concentration of one of the components of solid solution characterized by the correlation function [35]

$$\ll \xi(\mathbf{r}_1)\xi(\mathbf{r}_2) \gg = Nx(1-x)\delta(\mathbf{r}_1 - \mathbf{r}_2) \tag{18}$$

Further we estimate the criterion of applicability of the Born approximation using the method given in [35]. The mean number of type-A atoms in the volume R^2d (d is the QW width) is xNR^2d. The typical surplus number is of the order of $(xNR^2d)^{1/2}$. Hence the type-A atoms share variation is

$$\delta x = \frac{(xNR^2d)^{1/2}}{xNR^2d} \tag{19}$$

Thereby the exciton potential well is

$$V = \alpha \frac{(xNR^2d)^{1/2}}{xNR^2d} \tag{20}$$

($\alpha = \alpha_e - \alpha_h$) arises in the volume R^2d. For $\alpha_e = \alpha_h$ the 'exciton well' is equal to zero. This means that in the last case the interaction (17) does not produce a change in the total exciton energy. An analogous situation arises also for direct exciton scattering by ionized impurities (Sec. V). The Born approximation is valid under the condition $V \ll \hbar^2/M_{exc}R^2$, i.e.

$$\frac{\hbar^3}{RM_{exc}} \gg \alpha\sqrt{\frac{x}{Nd}} \tag{21}$$

The matrix element (17) for transitions between states $(n = m = 0, \mathbf{P})$ and $(n = m = 0, \mathbf{P}')$ has the form

$$< \mathbf{P}' \mid V(\mathbf{r}_e, \mathbf{r}_h) \mid \mathbf{P} > = \frac{1}{S}\exp\left(-(\mathbf{P}' - \mathbf{P})^2 l^2/4\hbar^2\right)\xi(\mathbf{P}' - \mathbf{P})$$

$$\left(\alpha_e \exp\left(il^2\mathbf{H}[\mathbf{P}, \mathbf{P}']/2\hbar^2 H\right) - \alpha_h \exp\left(-il^2\mathbf{H}[\mathbf{P}, \mathbf{P}']/2\hbar^2 H\right)\right) \tag{22}$$

Then

$$\ll |< \mathbf{P}' \mid V(\mathbf{r}_e, \mathbf{r}_h) \mid \mathbf{P} >|^2 \gg =$$

$$\frac{g}{S}\left(\alpha_e^2 + \alpha_h^2 - 2\alpha_e\alpha_h \cos\left(l^2[\mathbf{P}, \mathbf{P}']/\hbar^2\right)\right)\exp\left(-(\mathbf{P}' - \mathbf{P})^2 l^2/2\hbar^2\right), \tag{23}$$

where $g = Nx(1-x)$.

Using Eqs. (23) and (7) we obtain the transport relaxation time

$$\tau_{cf}^{-1}(P) = \frac{g}{\hbar^3}\left|\frac{\partial \mathcal{E}(P)}{\partial P^2}\right|^{-1}\exp(-P^2 l^2/\hbar^2)$$

$$\left(\frac{\alpha_e^2 + \alpha_h^2}{2}\left(I_0(P^2 l^2/\hbar^2) - I_1(P^2 l^2/\hbar^2)\right) + \alpha_e\alpha_h\left(\frac{P^2 l^2}{\hbar^2} - 1\right)\right) \tag{24}$$

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The function τ_{cf} is *nonmonotonous* (Fig. 3 a,b). Such behavior is due to *interplay between the maximum* of $\partial \mathcal{E}/\partial P^2$ and the *minimum* of the rest part of (24). With α_h/α_e increasing the minimum of $\tau_{cf}(P)$ appears at $P \neq 0$. Further increase of the parameter α_h/α_e leads to the minimum disappearing.

The asymptotics of expression (24) are

$$\tau_{cf}(P) = \frac{\hbar e^2 l \sqrt{\pi}}{2^{3/2} g \epsilon} \begin{cases} (\alpha_e - \alpha_h)^{-2}, & P = 0, \\ 8\pi(\alpha_e^2 + \alpha_h^2)^{-1}, & Pl/\hbar \gg 1 \end{cases} \tag{25}$$

Taking into account values of $\alpha_e - \alpha_h$ (see [35] and references therein) we obtain $\tau_{cf} \sim 10^{-13} - 10^{-11}$c. Therefore one can conclude that in sufficiently narrow QWs the magnetoexciton transport relaxation time defiened by exciton scattering on composition fluctuations have the same order as the relaxation time for scattering on IFR.

Using Eqs. (8) and (24) we obtain the magnetoexciton mean free path as function of P (Fig. 4 a,b). For sufficiently small values of the parameter α_e/α_h it has a maximum at $P \neq 0$ analogously to the case of magnetoexciton IFR scattering. With α_e/α_h increasing the second maximum appears and then disappears. The value λ_{max} decreases with increasing H. It indicates that in strong magnetic fields excitons may be localized in random fields of composite fluctuations.

IV. INDIRECT MAGNETOEXCITON IFR SCATTERING.

The scattering potential of a magnetoexciton with spatially separated e and h has the form

$$V(\mathbf{r}_e, \mathbf{r}_h) = \alpha_e(\xi_1(\mathbf{r}_e) - \xi_2(\mathbf{r}_e)) + \alpha_h(\xi_3(\mathbf{r}_h) - \xi_4(\mathbf{r}_h)), \tag{26}$$

where ξ_1, ξ_2 (ξ_3, ξ_4) are e(h) QW width fluctuations on the lower and upper interfaces, $\alpha_{e,h} = \frac{\partial E_{e,h}^{(0)}}{\partial d_{e,h}}$, $d_{e,h}$ are e and h QW widths. As before we assume that ξ_i in different interfaces are statistically independent and have Gaussian autocorrelation functions (2) on the same interface. This is possible under the condition that the interwell separation D between e and h QWs is larger than the amplitudes Δ of fluctuations on the nearest interfaces of e and h QWs. The opposite case $D \leq \Delta$ can be realized in *double* QWs and for $D = 0$ it is considered in [27].

For the case of interest we have

$$\ll |< \mathbf{P}' \mid V(r_e, r_h) \mid \mathbf{P} >|^2 \gg = \frac{\pi}{2S} \left\{ \alpha_e^2 \sum_{i=1,2} \exp\left(-(\mathbf{P}' - \mathbf{P})^2 l^2 \beta_i/2\hbar^2\right) (\Delta_i \Lambda_i)^2 + \right.$$

$$\left. \alpha_h^2 \sum_{i=3,4} \exp\left(-(\mathbf{P}' - \mathbf{P})^2 l^2 \beta_i/2\hbar^2\right) (\Delta_i \Lambda_i)^2 \right\} \tag{27}$$

The relaxation time of indirect magnetoexcitons is obtained from Eqs. (27) and (7) in the form

$$\tau^{-1}(P) = \tau_e^{-1}(P) + \tau_h^{-1}(P) , \tag{28}$$

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where

$$\tau_{e,h}^{-1}(P) = \frac{\pi}{4\hbar^3} \left| \frac{\partial \mathcal{E}(P,D)}{\partial P^2} \right|^{-1} \alpha_{e,h}^2 \sum_{i=1,2(3,4)} (\Delta_i \Lambda_i)^2 \exp(-P^2 l^2 \beta_i/\hbar^2)$$

$$\left(I_0(P^2 l^2 \beta_i/\hbar^2) - I_1(P^2 l^2 \beta_i/\hbar^2) \right), \tag{29}$$

$\mathcal{E}(P,D)$ is the dispersion law of an indirect magnetoexciton in the state with quantum numbers $n = m = 0$ [12]

$$\mathcal{E}(\mathcal{P},\mathcal{D}) = \frac{1}{2}\hbar\omega_c - \frac{e^2}{\epsilon l}\frac{1}{\sqrt{2}} f(\mathcal{D},\mathcal{P}) \tag{30}$$

$$f(\mathcal{P},\mathcal{D}) = \sum_{k=0}^{\infty} P_{2k}(g) \frac{(-1)^k (2k)!}{2^{2k}(k!)^2} \left\{ \left(\frac{2}{\mathcal{D}^2 + \mathcal{P}^2} \right)^{k+1/2} \gamma \left(k+1, \frac{\mathcal{D}^2 + \mathcal{P}^2}{2} \right) + \right.$$

$$\left. \left(\frac{\mathcal{D}^2 + \mathcal{P}^2}{2} \right)^k \Gamma \left(-k + \frac{1}{2}, \frac{\mathcal{D}^2 + \mathcal{P}^2}{2} \right) \right\}, \tag{31}$$

where $\mathcal{D} = D/l$, $\mathcal{P} = Pl/\hbar$ $g = \left(1 + \left(\frac{\mathcal{P}}{\mathcal{D}} \right)^2 \right)^{-1/2}$, $\Gamma(a,x)$ is the complete gamma-function, $\gamma(a,x)$ is the incomplete gamma-function, $P_k(x)$ is the Legendre polynominal [36]. The transport relaxation time of an indirect magnetoexciton with an electron in AlAs QW and a hole in GaAs QW is presented in Fig. 1. The parameters of e and h QWs coincide with the parameters used in Sec. 2 for transport relaxation time calculation of direct magnetoexciton. The dependence of $\tau(P,D)$ on P is nonmonotonous. With increasing D/l the value of maximum τ_{max} decreases due to decreasing indirect magnetoexciton group velocity. For D/l larger than some critical value the relaxation time $\tau(P,D)$ became a monotonously increasing function of P (Fig. 1). For sufficiently large D we expect that a minimum of $\tau(P,D)$ appears. But for large interwell separation our results may not be valid due to (5,6).

For a small magnetic momentum $Pl/\hbar \ll 1$ the relaxation time is

$$\tau(0,D) = \frac{2\hbar^3}{\pi} \frac{1}{M(\mathcal{D})} \left(\alpha_e^2 \sum_{i=1,2} (\Delta_i \Lambda_i)^2 + \alpha_h^2 \sum_{i=3,4} (\Delta_i \Lambda_i)^2 \right)^{-1}, \tag{32}$$

where

$$M(\mathcal{D}) = \frac{M_o}{(1 + \mathcal{D}^2) e^{\frac{\mathcal{D}^2}{2}} erfc \left(\frac{\mathcal{D}}{\sqrt{2}} \right) - \mathcal{D}\sqrt{\frac{2}{\pi}}} \tag{33}$$

is the indirect magnetoexciton effective mass in the state with quantum numbers $n = m = 0$ (see [12]), $M_o = \frac{2^{3/2} \epsilon \hbar^2}{e^2 l \sqrt{\pi}}$ is the direct magnetoexciton effective mass in the lowest Landau level [11].

With increasing magnetic field the relaxation time (32) decreases as $1/\sqrt{H}$ at $D \ll l$ and as $1/H^2$ at $D \gg l$ in agreement with experimentally observed data [4]. For $Pl/\hbar \gg 1$

and $PD/\hbar \gg 1$ the magnetoexciton dispersion law (30) is $\mathcal{E} \approx 0.5\hbar\omega_c - e^2\hbar/\epsilon Pl^2$. Hence for large magnetic momentum we obtain

$$\tau \approx \frac{4\sqrt{2}\hbar e^2 l}{\epsilon\sqrt{\pi}} \left(\alpha_e^2 \sum_{i=1,2} \frac{(\Delta_i\Lambda_i)^2}{\beta_i^{3/2}} + \alpha_h^2 \sum_{i=3,4} \frac{(\Delta_i\Lambda_i)^2}{\beta_i^{3/2}} \right)^{-1} \tag{34}$$

Expression (34) decreases as $1/\sqrt{H}$ and is independent of P and D.

It is interesting to note that $\tau(P,D)$ and $\lambda(P,D)$ may have several extrema for magnetoexcitons in the excited states due to nonmonotonous behavior of $\mathcal{E}_{nm}(P,D)$ [11,12]. Magnetoexciton dispersion laws calculated in [11,12] have $2n+2$ saddle points in the states with quantum numbers $(m \neq 0, n)$ and $2n+1$ saddle points in the case $(m=0,n)$. *Each saddle point corresponds to the maximum of the function $|V_g|$*. With D/l increasing the maximum of $|V_g|$ decreases and dissapears at some critical D/l [12]. This variation may lead to the existences of *several maxima* of $\tau(P)$ *which disappear with increasing D* (nonmonotonous behavior of interaction matrix elements must also be take into account).

The mean free path of indirect magnetoexciton is shown in Fig. 2. The value of λ_{max} decreases with increasing D/l. If $P=0$ we get $\lambda(0,D) = 0$ because $v(0,D) = 0$. If $Pl/\hbar \gg 1$ the mean free path is $\lambda \sim \frac{1}{P^2}$. With H increasing λ_{max} increases.

V. IONIZED IMPURITIES IN CQW

Here we consider the situation when ionized Coulomb impurities q_1 and q_2 are randomly distributed in both QWs with the corresponding concentrations n_1 and n_2. The interaction of excitons with impurities in CQW is described by the potential energy:

$$V(\mathbf{r}_e, \mathbf{r}_h) = V_e(\mathbf{r}_e) + V_h(\mathbf{r}_h) , \tag{35}$$

where

$$V_e(\mathbf{r}_e) = \sum_i \frac{-eq_1}{\epsilon|\mathbf{r}_e - \mathbf{r}_i|} + \sum_j \frac{-eq_2}{\epsilon\sqrt{(\mathbf{r}_e - \mathbf{r}_j)^2 + D^2}} \tag{36}$$

$$V_h(\mathbf{r}_h) = \sum_i \frac{eq_1}{\epsilon\sqrt{(\mathbf{r}_h - \mathbf{r}_i)^2 + D^2}} + \sum_j \frac{eq_2}{\epsilon|\mathbf{r}_h - \mathbf{r}_j|} \tag{37}$$

The impurities q_1 and q_2 occupy places with coordinates $\mathbf{r}_i$ and $\mathbf{r}_j$ in CQW. For $D=0$ all impurities are located in one QW. The averaged squared matrix element of exciton-impurity interaction (35) has the following form

$$\ll |< \mathbf{P}'|V(\mathbf{r}_e, \mathbf{r}_h)|\mathbf{P} >|^2 \gg = \exp\left(-\frac{(\mathbf{P}' - \mathbf{P})^2 l^2}{2\hbar^2}\right) \left(\frac{2\pi\hbar e}{\mathbf{P}' - \mathbf{P}}\right)^2 \frac{(n_1 q_1^2 + n_2 q_2^2)}{S\epsilon^2}$$

$$\left\{ 1 + \exp\left(-\frac{2D|\mathbf{P}' - \mathbf{P}|}{\hbar}\right) - 2\exp\left(-\frac{D|\mathbf{P}' - \mathbf{P}|}{\hbar}\right) \cos\left(\frac{[\mathbf{P}, \mathbf{P}']l^2}{\hbar^2}\right) \right\} \tag{38}$$

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Using Eqs. (7), (38), we get the inverse relaxation time in the form (Fig. 5)

$$\frac{1}{\tau_i(P,D)} = \frac{\pi}{2\hbar} \left(\frac{e}{\epsilon P}\right)^2 \left|\frac{\partial \mathcal{E}(P,D)}{\partial P^2}\right|^{-1} \exp\left(-\frac{P^2 l^2}{\hbar^2}\right) (n_1 q_1^2 + n_2 q_2^2) \int\limits_0^{2\pi} \exp\left(\frac{P^2 l^2 \cos\phi}{\hbar^2}\right)$$

$$\left(1 + \exp\left(-\frac{2PD\sqrt{2-2\cos\phi}}{\hbar}\right) - 2\exp\left(-\frac{PD\sqrt{2-2\cos\phi}}{\hbar}\right)\cos\left(\frac{P^2 l^2}{\hbar^2}\sin\phi\right)\right) d\phi \quad (39)$$

For $P = 0$ the transport relaxation time is

$$\tau_i(0,D) = \frac{\hbar^3}{4\pi^2} \frac{\epsilon^2}{e^2 D^2 M(\mathcal{D})} \frac{1}{(n_1 q_1^2 + n_2 q_2^2)} \quad (40)$$

The divergence in Eq. (40) at small P and D arises due to the use of unscreened Coulomb potential. The relaxation time $\tau_i(0,D)$ decreases with increasing H because of increasing magnetoexciton effective mass (33). For $Pl/\hbar \gg 1$ and $PD/\hbar \gg 1$ the relaxation time (39) in asymptotic approximation does not depend on P, D:

$$\tau_i = \frac{\hbar\epsilon}{\sqrt{2}\pi^{3/2} l(n_1 q_1^2 + n_2 q_2^2)} \quad (41)$$

and increases as $\sqrt{H}$. The mean free path of a magnetoexciton in the presence of impurities in CQW is shown in Fig. 6. The peak of $\lambda_i(P,D)$ decreases with increasing D/l. The calculated value of the mean free path $\lambda_i(P,D)$ controlled by ionized impurities is greater for the same P and D than $\lambda(P,D)$ controlled by interface roughness scattering (calculated in Sec. II and Sec. IV).

VI. CONCLUSIONS

In the present paper low temperature transport properties of direct and indirect magnetoexcitons in CQW are considered. The transport relaxation time and mean free path of magnetoexcitons are calculated in Born approximation for various scattering processes. In the case $Pl/\hbar \ll 1$ the transport of direct magnetoexcitons is found to be limited by the interface roughness scattering or composition fluctuations because of $\tau_i \to \infty$. The transport relaxation time of direct magnetoexciton scattering on composite fluctuations depends nonmonotonously on P. The transport relaxation times τ, τ_i are nonmonotonous functions of P for parameter D/l smaller than some critical value. The relaxation time $\tau(P,D)$ has a maximum at $Pl \approx 2.6/\hbar$ which decreases with increasing D and disappears at D greater than some critical value. The maximal time τ_{max} increases with increasing magnetic field strength and shifts to larger magnetic momenta. For large D/l the quantities τ and τ_i increase monotonously with P. The mean free path of direct and indirect magnetoexcitons is a nonmonotonous function of P. It has a maximum at $P \neq 0$ which decreases with D. For H increasing λ_{max} increases in the case of IFR scattering and decreases due to scattering on local composition fluctuations.

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Captions to Figures for Article
"Transport of magnetoexcitons in single and coupled quantum wells"
by Yu. . Lozovik and . . Ruvinsky

Fig. 1. Transport relaxation time of direct (dots line) and indirect (solid line) magnetoexciton scattering on interface terraces in coupled quantum wells at $H = 2 \cdot 10^5$Oe as the function of magnetic momentum P. Lines 1,2,3,4,5 correspond to interwell sepration $D/l = 0.1; 0.5; 1; 2; 3$. P is given in units $\hbar/l$.

Fig. 2. Mean free path of direct (broken line) and indirect (solid lines) magnetoexciton scattering on interface terraces in coupled quantum wells at $H = 2 \cdot 10^5$Oe as a function of magnetic momentum P. Lines 1,2,3 correspond to $D/l = 0.1; 0.5; 1$.

Fig. 3. Transport relaxation time of direct magnetoexciton scattering on composition fluctuations at $H = 2 \cdot 10^5$Oe as a function of magnetic momentum P.
a) Lines 1,2,3,4,5 correspond to $\alpha_h/\alpha_e = 0; 0.25; 0.5; 0.75; 1$; τ_{cf} is given in units $e^2 \hbar l / g \alpha_e^2 \epsilon$, P is given in units $\hbar/l$.
b) Lines 1,2,3,4 correspond to $\alpha_h/\alpha_e = 1.75; 2; 4; 5$.

Fig. 4.
Mean free path of direct magnetoexciton scattering on composition fluctuations at $H = 2 \cdot 10^5$Oe as a function of magnetic momentum P.
a) Lines 1,2,3,4,5,6 correspond to $\alpha_h/\alpha_e = 0; 0.25; 0.5; 0.75; 1.25; 1.5$; λ_{cf} is given in units $e^4 l / g \alpha_e^2 \epsilon^2$.
b) Lines 1,2,3,4,5 correspond to $\alpha_h/\alpha_e = 1.75; 2; 4; 5; 6$.

Fig. 5. The inverse transport relaxation time of direct (broken line) and indirect (solid lines) magnetoexciton scattering on ionized impurities in coupled quantum wells at $H = 2 \cdot 10^5$Oe as a function of magnetic momentum P. Lines 1,2,3,4 correspond to $D/l = 0.1; 0.3; 0.5; 1$; impurity concentrations $c_1 = c_2 = 10^{10}$cm^{-2}.

Fig. 6. Mean free path of direct (broken line) and indirect (solid lines) magnetoexciton scattering on ionized impurities in coupled quantum wells at $H = 2 \cdot 10^5$Oe as a function of magnetic momentum P. Lines 1,2,3 correspond to $D/l = 0.1; 0.5; 1$; impurity concentrations $c_1 = c_2 = 10^{10}$cm^{-2}.